Another Monte Carlo Example.

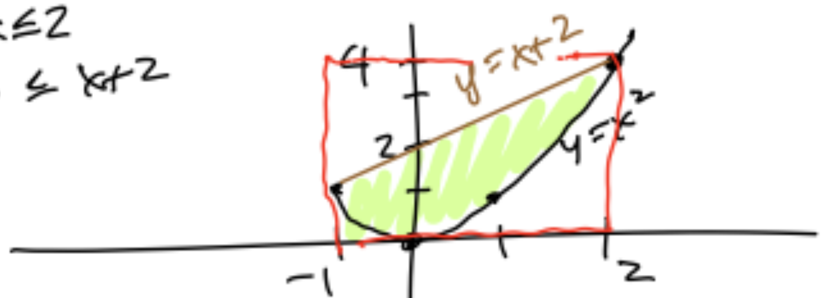
Estimate $\displaystyle\int_{-1}^{2}\int_{x^2}^{x+2} e^{-x^2+y}\,dy\,dx.$

Region of integration

$$\int_{x=-1}^{2}\left(\int_{y=x^2}^{x+2} e^{-x^2+y}\,dy\right)dx$$

$-1 \le x \le 2$

$x^2 \le y \le x+2$



We will: ① choose random

x in $[-1, 2]$

y in $[0, 4]$

② Check if $x^2 \le y$ and $y \le x+2$

(2B) if so, evaluate $f(x,y) = e^{-x^2+y}$ on the point, add to sum.

Do this a lot.

→ (Take avg of 2B's.)

· Area of Region → ≈ integral

integral $\approx \dfrac{\text{sum}}{N} \cdot (\text{area})$.

N ← # pts inside region found

$$\text{Area} = \int_{-1}^{2} (x+2) - x^2 \, dx$$

$$\frac{x^2}{2} + 2x - \frac{x^3}{3} \Big|_{-1}^{2}$$

$$= 2 + 4 - \frac{8}{3} - \Big(\underbrace{\frac{1}{2} - 2}_{-\frac{3}{2}} + \frac{1}{3}\Big)$$

$$= 6 \underbrace{- \frac{8}{3} + \frac{3}{2} - \frac{1}{3}}_{-3} = 3 + \frac{3}{2} = \boxed{4.5}$$

Sagemath → calculate use random #s.

We got with $N = 1{,}000{,}000$

Integral ≈ 13.2295

(actually $13.247\ldots$)

New topic

Solving equations using numerical methods.

Example: Solve the equation

$$x - 1.5 = \ln(x).$$

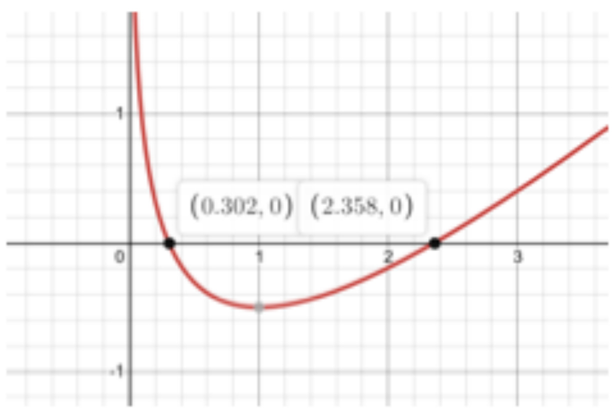
Looking at the graph

from the picture, we should have 2 solutions!

Using ordinary algebraic methods, we can't solve for them,

① Convert to solving for roots (zeros) of the function $f(x) = x - 1.5 - \ln(x)$

② To solve, we just need to find out where the function is zero.



How can we solve for the zeros?

There are many different algorithms.

<u>Simplest</u>: Method of bisection.

Let's try to find the first root.

Guess $\exists$ zero of $f(x) = x - 1.5 - \ln(x)$

between $x = 1/4$ & $x = 1$.

check $f(\frac{1}{4}) = \cdots > 0$

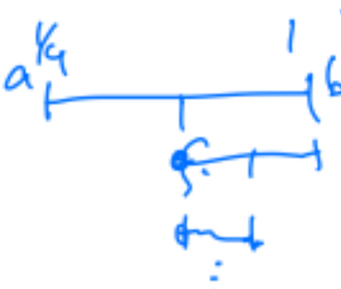
$f(1) = \_\_\_ < 0$

$a = \frac{1}{4}, \quad b = 1$

<u>next guess</u>: Try $c = \frac{5}{8} = \frac{a+b}{2}$.

$$\text{if} \begin{cases} c = 0, \text{ done} \\ c > 0, \text{ change } a \text{ to } c, \text{ go again.} \\ c < 0, \text{ change } b \text{ to } c, \text{ go again.} \end{cases}$$

Keep doing that — get smaller & smaller interval where the zero is located



idea.